

الجذور المربعة

حلول التمارين

$$\begin{aligned} \sqrt{8^4} &= \sqrt{(8^2)^2} = 8^2 = 64 \quad , \quad \sqrt{144} = \sqrt{12^2} = 12 \quad , \quad \sqrt{81} = \sqrt{9^2} = 9 \quad (1) \\ \sqrt{10^{-6}} &= \sqrt{(10^{-3})^2} & \sqrt{\frac{49}{36}} &= \sqrt{\frac{7^2}{6^2}} & \sqrt{2^4 \times 5^2 \times 7^6} &= \sqrt{(2^2)^2 \times 5^2 \times (7^3)^2} \\ &= 10^{-3} & &= \sqrt{\left(\frac{7}{6}\right)^2} & &= \sqrt{(2^2 \times 5 \times 7^3)^2} \\ & & &= \frac{7}{6} & &= 2^2 \times 5 \times 7^3 \end{aligned}$$

$$\begin{aligned} \sqrt{3} \times \sqrt{12} &= \sqrt{3 \times 12} & \sqrt{0,0025} &= \sqrt{25 \times 10^{-4}} \\ &= \sqrt{36} & &= \sqrt{5^2 \times (10^{-2})^2} \\ &= 6 & &= \sqrt{(5 \times 10^{-2})^2} \\ & & &= 5 \times 10^{-2} \\ & & &= 0,05 \end{aligned}$$

$$\frac{\sqrt{24}}{\sqrt{54}} = \sqrt{\frac{24}{54}} = \sqrt{\frac{6 \times 4}{6 \times 9}} = \sqrt{\frac{4}{9}} = \sqrt{\left(\frac{2}{3}\right)^2} = \frac{2}{3}$$

$$\begin{aligned} (3\sqrt{5})^3 &= 3^3 (\sqrt{5})^3 & , \quad (-\sqrt{11})^2 &= (\sqrt{11})^2 & , \quad (\sqrt{5})^2 &= 5 \quad (2) \\ &= 27 (\sqrt{5})^2 \sqrt{5} & &= 11 & & \\ &= 27 \times 5 \times \sqrt{5} & & & & \\ &= 135\sqrt{5} & & & & \end{aligned}$$

$$\left(\frac{-\sqrt{7}}{4}\right)^2 = \left(\frac{\sqrt{7}}{4}\right)^2 = \frac{7}{16}$$

$$\sqrt{2^2 \times 5^6} = 2 \times 5^3 \quad , \quad \sqrt{\frac{64}{25}} = \frac{8}{5} \quad , \quad \sqrt{49} = 7 \quad (3)$$

$$\begin{aligned} \sqrt{\frac{4a^6 b^8}{c^{10}}} &= \sqrt{\left(\frac{2a^3 b^4}{c^5}\right)^2} & \sqrt{9a^4 b^6 c^2} &= \sqrt{(3a^2 b^3 c)^2} & \sqrt{a^2 b^4} &= \sqrt{(ab^2)^2} \quad (4) \\ &= \frac{2a^3 b^4}{c^5} & &= 3a^2 b^3 c & &= ab^2 \end{aligned}$$

$$\sqrt{63} - \sqrt{112} + \sqrt{700} = \sqrt{7 \times 9} - \sqrt{7 \times 16} + \sqrt{7 \times 100} \quad (5)$$

$$\begin{aligned}
 &= \sqrt{9} \times \sqrt{7} - \sqrt{16} \times \sqrt{7} + \sqrt{100} \times \sqrt{7} \\
 &= 3\sqrt{7} - 4\sqrt{7} + 10\sqrt{7} \\
 &= 9\sqrt{7}
 \end{aligned}$$

$$\begin{aligned}
 \sqrt{99} - 10\sqrt{1100} - 6\sqrt{396} &= \sqrt{9 \times 11} - 10\sqrt{100 \times 11} - 6\sqrt{36 \times 11} \\
 &= \sqrt{9} \times \sqrt{11} - 10\sqrt{100} \times \sqrt{11} - 6\sqrt{36} \times \sqrt{11} \\
 &= 3\sqrt{11} - 10 \times 10\sqrt{11} - 6 \times 6\sqrt{11} \\
 &= 3\sqrt{11} - 100\sqrt{11} - 36\sqrt{11} \\
 &= -133\sqrt{11}
 \end{aligned}$$

$$\begin{aligned}
 \frac{3}{4}\sqrt{48} - 0,5\sqrt{108} &= \frac{3}{4}\sqrt{16 \times 3} - 0,5\sqrt{36 \times 3} \\
 &= \frac{3}{4} \times 4\sqrt{3} - 0,5 \times 6\sqrt{3} \\
 &= 3\sqrt{3} - 3\sqrt{3} \\
 &= 0
 \end{aligned}$$

$$\begin{aligned}
 \sqrt{\frac{7}{3}} + 4\sqrt{\frac{63}{75}} - 2\sqrt{\frac{28}{27}} &= \sqrt{\frac{7}{3}} + 4\sqrt{\frac{9 \times 7}{25 \times 3}} - 2\sqrt{\frac{4 \times 7}{9 \times 3}} \\
 &= \sqrt{\frac{7}{3}} + 4 \times \frac{3}{5} \times \sqrt{\frac{7}{3}} - 2 \times \frac{2}{3} \times \sqrt{\frac{7}{3}} \\
 &= \sqrt{\frac{7}{3}} + \frac{12}{5} \sqrt{\frac{7}{3}} - \frac{4}{3} \sqrt{\frac{7}{3}} \\
 &= \left(1 + \frac{12}{5} - \frac{4}{3}\right) \times \sqrt{\frac{7}{3}} \\
 &= \left(\frac{15 + 36 - 20}{15}\right) \times \sqrt{\frac{7}{3}} \\
 &= \frac{31}{15} \sqrt{\frac{7}{3}}
 \end{aligned}$$

$$\begin{aligned}
 (3\sqrt{2} - 2)^2 &= (3\sqrt{2})^2 - 2 \times 3\sqrt{2} \times 2 + 2^2 \\
 &= 9 \times 2 - 12\sqrt{2} + 4 \\
 &= 18 - 12\sqrt{2} + 4 \\
 &= 22 - 12\sqrt{2}
 \end{aligned}$$

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$$\begin{aligned}
 (\sqrt{15} - \sqrt{30})(\sqrt{15} + \sqrt{30}) &= (\sqrt{15})^2 - (\sqrt{30})^2 \\
 &= 15 - 30 \\
 &= -15
 \end{aligned}$$

$$\begin{aligned}
 (2 + \sqrt{2} + \sqrt{3})(2 + \sqrt{2} - \sqrt{3}) &= [(2 + \sqrt{2}) + \sqrt{3}] \times [(2 + \sqrt{2}) - \sqrt{3}] \\
 &= (2 + \sqrt{2})^2 - (\sqrt{3})^2
 \end{aligned}$$

$$\begin{aligned}
 &= 2^2 + 2 \times 2 \times \sqrt{2} + (\sqrt{2})^2 - 3 \\
 &= 4 - 4\sqrt{2} + 2 - 3 \\
 &= 3 - 4\sqrt{2}
 \end{aligned}$$

$$\begin{aligned}
 29 - 12\sqrt{5} &= (3 - 2\sqrt{5})^2 \quad , \quad 9 - 4\sqrt{5} = (2 - \sqrt{5})^2 \quad , \quad 4 + 2\sqrt{3} = (1 + \sqrt{3})^2 \quad (7) \\
 \mathbf{a + 4\sqrt{a} + 4} &= (\mathbf{\sqrt{a} + 2})^2
 \end{aligned}$$

$$\begin{aligned}
 \sqrt{\mathbf{a + 4\sqrt{a} + 4}} &= \sqrt{(\mathbf{\sqrt{a} + 2})^2} \quad , \quad \sqrt{9 - 4\sqrt{5}} = \sqrt{(2 - \sqrt{5})^2} \quad , \quad \sqrt{4 + 2\sqrt{3}} = \sqrt{(1 + \sqrt{3})^2} \quad (8) \\
 &= \mathbf{\sqrt{a} + 2} \quad , \quad = \sqrt{(\sqrt{5} - 2)^2} \quad = 1 + \sqrt{3} \\
 &= \sqrt{5} - 2 \\
 &(\sqrt{5} - 2 > 0 \quad \text{لاحظ أن})
 \end{aligned}$$

$$\begin{aligned}
 \sqrt{11 + 6\sqrt{2}} &= \sqrt{(\sqrt{2} + 3)^2} \quad , \quad \sqrt{21 + 4\sqrt{5}} = \sqrt{(2\sqrt{5} + 1)^2} \quad , \quad \sqrt{7 + 2\sqrt{10}} = \sqrt{(\sqrt{5} + \sqrt{2})^2} \quad (9) \\
 &= \sqrt{2} + 3 \quad = 2\sqrt{5} + 1 \quad = \sqrt{5} + \sqrt{2}
 \end{aligned}$$

$$\begin{aligned}
 \frac{2}{1 + \sqrt{5}} &= \frac{2(\sqrt{5} - 1)}{(\sqrt{5} + 1)(\sqrt{5} - 1)} & \frac{10}{\sqrt{5}} &= \frac{10\sqrt{5}}{(\sqrt{5})^2} & (10) \\
 &= \frac{2(\sqrt{5} - 1)}{(\sqrt{5})^2 - 1^2} & &= \frac{10\sqrt{5}}{5} \\
 &= \frac{2(\sqrt{5} - 1)}{5 - 1} & &= 2\sqrt{5} \\
 &= \frac{2(\sqrt{5} - 1)}{4} \\
 &= \frac{\sqrt{5} - 1}{2}
 \end{aligned}$$

$$\begin{aligned}
 \frac{14}{3\sqrt{3} + 2\sqrt{5}} &= \frac{14(3\sqrt{3} - 2\sqrt{5})}{(3\sqrt{3} + 2\sqrt{5})(3\sqrt{3} - 2\sqrt{5})} & \frac{\sqrt{2} + \sqrt{3}}{\sqrt{2} - \sqrt{3}} &= \frac{(\sqrt{2} + \sqrt{3})(\sqrt{2} + \sqrt{3})}{(\sqrt{2} - \sqrt{3})(\sqrt{2} + \sqrt{3})} \\
 &= \frac{14(3\sqrt{3} - 2\sqrt{5})}{(3\sqrt{3})^2 - (2\sqrt{5})^2} & &= \frac{(\sqrt{2} + \sqrt{3})^2}{(\sqrt{2})^2 - (\sqrt{3})^2} \\
 &= \frac{14(3\sqrt{3} - 2\sqrt{5})}{27 - 20} & &= \frac{(\sqrt{2})^2 + 2 \times \sqrt{2} \times \sqrt{3} + (\sqrt{3})^2}{2 - 3} \\
 &= \frac{14(3\sqrt{3} - 2\sqrt{5})}{7} & &= \frac{2 + 2\sqrt{6} + 3}{-1} \\
 &= 2(3\sqrt{3} - 2\sqrt{5}) = 6\sqrt{3} - 4\sqrt{5} & &= -5 - 2\sqrt{6} \\
 \frac{1}{\sqrt{\mathbf{a + b}} - \sqrt{\mathbf{a}}} &= \frac{1(\sqrt{\mathbf{a + b}} + \sqrt{\mathbf{a}})}{(\sqrt{\mathbf{a + b}} - \sqrt{\mathbf{a}})(\sqrt{\mathbf{a + b}} + \sqrt{\mathbf{a}})} & \frac{2\sqrt{\mathbf{a}}}{1 - \sqrt{\mathbf{a}}} &= \frac{2\sqrt{\mathbf{a}}(1 + \sqrt{\mathbf{a}})}{(1 - \sqrt{\mathbf{a}})(1 + \sqrt{\mathbf{a}})}
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{\sqrt{a+b} + \sqrt{a}}{(\sqrt{a+b})^2 - (\sqrt{a})^2} &= \frac{2\sqrt{a} + 2(\sqrt{a})^2}{1 - (\sqrt{a})^2} \\
 &= \frac{\sqrt{a+b} + \sqrt{a}}{a+b-a} &= \frac{2\sqrt{a} + 2a}{1-a} \\
 &= \frac{\sqrt{a+b} + \sqrt{a}}{b}
 \end{aligned}$$

$$\begin{aligned}
 \frac{\sqrt{2} + \sqrt{3} - \sqrt{5}}{\sqrt{2} - \sqrt{3} + \sqrt{5}} &= \frac{(\sqrt{2} + \sqrt{3} - \sqrt{5})((\sqrt{2} - \sqrt{3}) - \sqrt{5})}{((\sqrt{2} - \sqrt{3}) + \sqrt{5})((\sqrt{2} - \sqrt{3}) - \sqrt{5})} && \text{(11)} \\
 &= \frac{((\sqrt{2} - \sqrt{5}) + \sqrt{3})((\sqrt{2} - \sqrt{5}) - \sqrt{3})}{(\sqrt{2} - \sqrt{3})^2 - (\sqrt{5})^2} \\
 &= \frac{(\sqrt{2} - \sqrt{5})^2 - (\sqrt{3})^2}{2 + 3 - 2 \times \sqrt{2} \times \sqrt{3} - 5} \\
 &= \frac{2 + 5 - 2 \times \sqrt{2} \times \sqrt{3} - 3}{-2\sqrt{6}} \\
 &= \frac{4 - 2\sqrt{10}}{-2\sqrt{6}} \\
 &= \frac{(4 - 2\sqrt{10}) \times \sqrt{6}}{-2\sqrt{6} \times \sqrt{6}} \\
 &= \frac{4\sqrt{6} - 2\sqrt{60}}{-2 \times 6} \\
 &= \frac{4\sqrt{6} - 2\sqrt{60}}{-12} \\
 &= \frac{2\sqrt{60} - 4\sqrt{6}}{12} \\
 &= \frac{\sqrt{60} - 2\sqrt{6}}{6}
 \end{aligned}$$

$$\begin{aligned}
 \frac{1}{\sqrt{5} - \sqrt{3} + 2\sqrt{2}} &= \frac{1((\sqrt{5} - \sqrt{3}) - 2\sqrt{2})}{((\sqrt{5} - \sqrt{3}) + 2\sqrt{2})((\sqrt{5} - \sqrt{3}) - 2\sqrt{2})} \\
 &= \frac{\sqrt{5} - \sqrt{3} - 2\sqrt{2}}{(\sqrt{5} - \sqrt{3})^2 - (2\sqrt{2})^2} \\
 &= \frac{\sqrt{5} - \sqrt{3} - 2\sqrt{2}}{(\sqrt{5})^2 - 2 \times \sqrt{5} \times \sqrt{3} + (\sqrt{3})^2 - 8} \\
 &= \frac{\sqrt{5} - \sqrt{3} - 2\sqrt{2}}{5 - 2\sqrt{15} + 3 - 8} \\
 &= \frac{\sqrt{5} - \sqrt{3} - 2\sqrt{2}}{-2\sqrt{15}}
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{(\sqrt{5} - \sqrt{3} - 2\sqrt{2}) \times \sqrt{15}}{-2\sqrt{15} \times \sqrt{15}} \\
 &= \frac{\sqrt{5} \times \sqrt{15} - \sqrt{3} \times \sqrt{15} - 2\sqrt{2} \times \sqrt{15}}{-2 \times 15} \\
 &= \frac{\sqrt{5} \times \sqrt{5} \times \sqrt{3} - \sqrt{3} \times \sqrt{3} \times \sqrt{5} - 2\sqrt{30}}{-30} \\
 &= \frac{5\sqrt{3} - 3\sqrt{5} - 2\sqrt{30}}{-30} \\
 &= \frac{3\sqrt{5} + 2\sqrt{30} - 5\sqrt{3}}{30}
 \end{aligned}$$

$$\begin{aligned}
 \frac{1}{\sqrt{3} + \sqrt{2} - 1} + \frac{1}{\sqrt{3} + \sqrt{2} + 1} &= \frac{(\sqrt{3} + \sqrt{2} + 1) + (\sqrt{3} + \sqrt{2} - 1)}{(\sqrt{3} + \sqrt{2} + 1)(\sqrt{3} + \sqrt{2} - 1)} \\
 &= \frac{2\sqrt{3} + 2\sqrt{2}}{(\sqrt{3} + \sqrt{2})^2 - 1^2} \\
 &= \frac{2\sqrt{3} + 2\sqrt{2}}{3 + 2 + 2\sqrt{6} - 1} \\
 &= \frac{2\sqrt{3} + 2\sqrt{2}}{2\sqrt{6} + 4} \\
 &= \frac{2(\sqrt{3} + \sqrt{2})}{2(\sqrt{6} + 2)} \\
 &= \frac{(\sqrt{3} + \sqrt{2})(\sqrt{6} - 2)}{(\sqrt{6} + 2)(\sqrt{6} - 2)} \\
 &= \frac{\sqrt{18} - 2\sqrt{3} + \sqrt{12} - 2\sqrt{2}}{(\sqrt{6})^2 - 2^2} \\
 &= \frac{3\sqrt{2} - 2\sqrt{3} + 2\sqrt{3} - 2\sqrt{2}}{6 - 4} \\
 &= \frac{\sqrt{2}}{2}
 \end{aligned}$$

(12) لمقارنة $\frac{3}{\sqrt{3} + \sqrt{2}}$ و $\frac{1}{\sqrt{3} - \sqrt{2}}$

طريقة 1 : ندرس إشارة فرقهما:

$$\begin{aligned}
 \frac{3}{\sqrt{3} + \sqrt{2}} - \frac{1}{\sqrt{3} - \sqrt{2}} &= \frac{3(\sqrt{3} - \sqrt{2}) - (\sqrt{3} + \sqrt{2})}{(\sqrt{3} + \sqrt{2})(\sqrt{3} - \sqrt{2})} \\
 &= \frac{3\sqrt{3} - 3\sqrt{2} - \sqrt{3} - \sqrt{2}}{(\sqrt{3})^2 - (\sqrt{2})^2} \\
 &= \frac{2\sqrt{3} - 4\sqrt{2}}{3 - 2}
 \end{aligned}$$

$$= \frac{2(\sqrt{3} - 2\sqrt{2})}{1}$$

$$= 2(\sqrt{3} - 2\sqrt{2})$$

ولدينا $(\sqrt{3})^2 = 3$ و $(2\sqrt{2})^2 = 8$ إذن $\sqrt{3} < 2\sqrt{2}$ ومنه $\sqrt{3} - 2\sqrt{2} < 0$
و بالتالي $\frac{1}{\sqrt{3} + \sqrt{2}} < \frac{1}{\sqrt{3} - \sqrt{2}}$

طريقة 2 :

نجعل المقام جذريا أولا ثم نقارن

$$\frac{1}{\sqrt{3} - \sqrt{2}} = \frac{\sqrt{3} + \sqrt{2}}{(\sqrt{3} - \sqrt{2})(\sqrt{3} + \sqrt{2})}$$

$$= \frac{\sqrt{3} + \sqrt{2}}{3 - 2}$$

$$= \sqrt{3} + \sqrt{2}$$

$$\frac{3}{\sqrt{3} - \sqrt{2}} = \frac{3(\sqrt{3} - \sqrt{2})}{(\sqrt{3} - \sqrt{2})(\sqrt{3} + \sqrt{2})}$$

$$= \frac{3(\sqrt{3} - \sqrt{2})}{3 - 2}$$

$$= 3\sqrt{3} - 3\sqrt{2}$$

و نقارن $\sqrt{3} + \sqrt{2}$ و $3\sqrt{3} - 3\sqrt{2}$ بدراسة إشارة فرقهما مثل الطريقة 1 .

$$(\sqrt{a} + \sqrt{b})^2 - 4\sqrt{ab} = (\sqrt{a})^2 + 2 \times \sqrt{a} \times \sqrt{b} + (\sqrt{b})^2 - 4\sqrt{ab} \quad \text{لدينا } \mathbf{(13) - 1}$$

$$= a + 2\sqrt{ab} + b - 4\sqrt{ab}$$

$$= a - 2\sqrt{ab} + b$$

$$= (\sqrt{a} - \sqrt{b})^2$$

$$(\sqrt{a} + \sqrt{b})^2 \geq 4\sqrt{ab} \quad \text{إذن } (\sqrt{a} - \sqrt{b})^2 \geq 0$$

$$(\sqrt{a} + \sqrt{b})^2 - (\sqrt{a+b})^2 = a + 2\sqrt{ab} + b - (a+b) \quad \mathbf{- 2}$$

$$= 2\sqrt{ab}$$

$$(\sqrt{a} + \sqrt{b})^2 \geq (\sqrt{a+b})^2 \quad \text{إذن } 2\sqrt{ab} \geq 0$$

إذن يكون التساوي إذا كان $\sqrt{ab} = 0$ أي $ab = 0$ يعني $a = 0$ أو $b = 0$

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$$(\sqrt{18 + \sqrt{8}})^2 = 18 + \sqrt{8} \quad \text{لدينا}$$

$$= 18 + \sqrt{4 \times 2}$$

$$= 18 + 2\sqrt{2}$$

$$(\sqrt{9 + \sqrt{79}} + \sqrt{9 - \sqrt{79}})^2 = (\sqrt{9 + \sqrt{79}})^2 + 2 \times \sqrt{9 + \sqrt{79}} \times \sqrt{9 - \sqrt{79}} + (\sqrt{9 - \sqrt{79}})^2$$

$$\begin{aligned}
 &= 9 + \sqrt{79} + 2 \times \sqrt{9^2 - (\sqrt{79})^2} + 9 - \sqrt{79} \\
 &= 18 + 2\sqrt{81 - 79} \\
 &= 18 + 2\sqrt{2}
 \end{aligned}$$

و العددين $\sqrt{18 + \sqrt{8}}$ و $\sqrt{9 + \sqrt{79}} + \sqrt{9 - \sqrt{79}}$ موجبان إذن :

$$\sqrt{18 + \sqrt{8}} = \sqrt{9 + \sqrt{79}} + \sqrt{9 - \sqrt{79}}$$

و بنفس الطريقة

$$\left(\sqrt{6 + \sqrt{5}}\right)^2 = 6 + \sqrt{5}$$

$$\begin{aligned}
 \sqrt{\frac{6 + \sqrt{31}}{2}} + \sqrt{\frac{6 - \sqrt{31}}{2}} &= \left(\sqrt{\frac{6 + \sqrt{31}}{2}}\right)^2 + 2 \times \sqrt{\frac{6 + \sqrt{31}}{2} \times \frac{6 - \sqrt{31}}{2}} + \left(\sqrt{\frac{6 - \sqrt{31}}{2}}\right)^2 \\
 &= \frac{6 + \sqrt{31}}{2} + 2 \times \sqrt{\frac{6^2 - (\sqrt{31})^2}{4}} + \frac{6 - \sqrt{31}}{2} \\
 &= \frac{6 + \sqrt{31} + 2\sqrt{36 - 31} + 6 - \sqrt{31}}{2} \\
 &= \frac{12 + 2\sqrt{5}}{2} \\
 &= 6 + \sqrt{5}
 \end{aligned}$$

إذن : $\sqrt{6 + \sqrt{5}} = \sqrt{\frac{6 + \sqrt{31}}{2}} + \sqrt{\frac{6 - \sqrt{31}}{2}}$

$$\begin{aligned}
 \frac{a^2 + 2a + 2a\sqrt{b} + b + 2\sqrt{b}}{a^2 - a + a\sqrt{b} - \sqrt{b}} &= \frac{(a^2 + 2a\sqrt{b} + b) + (2a + 2\sqrt{b})}{(a^2 - a) + (a\sqrt{b} - \sqrt{b})} \quad (15) \\
 &= \frac{(a + \sqrt{b})^2 + 2(a + \sqrt{b})}{a(a - 1) + \sqrt{b}(a - 1)} \\
 &= \frac{(a + \sqrt{b})[(a + \sqrt{b}) + 2]}{(a + \sqrt{b})(a - 1)} \\
 &= \frac{a + \sqrt{b} + 2}{a - 1}
 \end{aligned}$$

$$\begin{aligned}
 \sqrt{a} \frac{\sqrt{1 + \frac{2b}{1 + b^2}} + \sqrt{1 - \frac{2b}{1 + b^2}}}{\sqrt{a + \frac{2ab}{1 + b^2}} - \sqrt{a - \frac{2ab}{1 + b^2}}} &= \sqrt{a} \frac{\sqrt{\frac{1 + b^2 + 2b}{1 + b^2}} + \sqrt{\frac{1 + b^2 - 2b}{1 + b^2}}}{\sqrt{\frac{a + ab^2 + 2ab}{1 + b^2}} - \sqrt{\frac{a + ab^2 - 2ab}{1 + b^2}}} \quad (16) \\
 &= \sqrt{a} \frac{\frac{\sqrt{(1 + b)^2}}{\sqrt{1 + b^2}} + \frac{\sqrt{(b - 1)^2}}{\sqrt{1 + b^2}}}{\frac{\sqrt{a(1 + b^2 + 2b)}}{\sqrt{1 + b^2}} - \frac{\sqrt{a(1 + b^2 - 2b)}}{\sqrt{1 + b^2}}}
 \end{aligned}$$

$$\begin{aligned}
 &= \sqrt{a} \frac{\frac{1+b}{\sqrt{1+b^2}} + \frac{b-1}{\sqrt{1+b^2}}}{\frac{\sqrt{a}\sqrt{(1+b)^2}}{\sqrt{1+b^2}} - \frac{\sqrt{a}\sqrt{(b-1)^2}}{\sqrt{1+b^2}}} \\
 &= \sqrt{a} \frac{\frac{1+b+b-1}{\sqrt{1+b^2}}}{\frac{\sqrt{a}(1+b) - \sqrt{a}(b-1)}{\sqrt{1+b^2}}} \\
 &= \sqrt{a} \frac{2b}{\frac{\sqrt{a}[(1+b) - (b-1)]}{\sqrt{1+b^2}}} \\
 &= \sqrt{a} \frac{2b}{\frac{\sqrt{a}(1+b-b+1)}{\sqrt{1+b^2}}} \\
 &= \frac{2b}{\sqrt{1+b^2}} \times \frac{\sqrt{1+b^2}}{2} \\
 &= b
 \end{aligned}$$

(17) لحساب A نجعل أولا المقام جذريا في كل حد

$$\begin{aligned}
 A &= \frac{1}{\sqrt{2}+1} + \frac{1}{\sqrt{3}+\sqrt{2}} + \dots + \frac{1}{\sqrt{100}+\sqrt{99}} \\
 &= \frac{\sqrt{2}-1}{(\sqrt{2}+1)(\sqrt{2}-1)} + \frac{\sqrt{3}-\sqrt{2}}{(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})} + \dots + \frac{\sqrt{100}-\sqrt{99}}{(\sqrt{100}+\sqrt{99})(\sqrt{100}-\sqrt{99})} \\
 &= \frac{\sqrt{2}-1}{2-1} + \frac{\sqrt{3}-\sqrt{2}}{3-2} + \dots + \frac{\sqrt{100}-\sqrt{99}}{100-99} \\
 &= \cancel{\sqrt{2}-1} + \cancel{\sqrt{3}-\sqrt{2}} + \dots + \cancel{\sqrt{100}-\sqrt{99}} \\
 &= -1 + \sqrt{100} \\
 &= -1 + 10 = 9
 \end{aligned}$$

إذن $A = 9$

(18) نلاحظ أولا أن AC هي أكبر هذه المسافات

إذن نقارن $AB+BC$ مع AC

لدينا

$$\begin{aligned}
 AB + BC &= \sqrt{343} + \sqrt{63} \\
 &= \sqrt{49 \times 7} + \sqrt{9 \times 7} \\
 &= 7\sqrt{7} + 3\sqrt{7} \\
 &= 10\sqrt{7}
 \end{aligned}$$

$$AC = \sqrt{700} = \sqrt{100 \times 7} = 10\sqrt{7}$$

و

$$AB + BC = AC$$

و بالتالي النقط A و B و C مستقيمة