



تمرين ①

لنحسب ما يلي :

$$\left(\frac{1}{\sqrt{11}}\right)^{-2} = (\sqrt{11})^2$$

$$= 11$$

$$\left(\frac{\sqrt{3}}{-\sqrt{5}}\right)^{-4} = \left(\frac{-\sqrt{5}}{\sqrt{3}}\right)^4$$

$$= \frac{\sqrt{5}^4}{\sqrt{3}^4}$$

$$= \frac{5^2}{3^2}$$

$$= \frac{25}{9}$$

$$\frac{\sqrt{5}^5}{\sqrt{5}^8} = \frac{\sqrt{5^4 \times 5}}{(\sqrt{5^4})^2}$$

$$= \frac{5^2 \sqrt{5}}{5^4}$$

$$= \frac{\sqrt{5}}{5^2}$$

$$= \frac{\sqrt{5}}{25}$$

$$(-\sqrt{6})^5 = -\sqrt{6}^5$$

$$= -\sqrt{6^4 \times 6}$$

$$= -\sqrt{6^4} \times \sqrt{6}$$

$$= -(\sqrt{6^2})^2 \times \sqrt{6}$$

$$= -6^2 \sqrt{6}$$

$$= -36\sqrt{6}$$

$$\left(\frac{1}{\sqrt{7}}\right)^3 = \frac{1}{\sqrt{7}^3}$$

$$= \frac{1}{\sqrt{7^2 \times 7}}$$

$$= \frac{1}{7\sqrt{7}}$$

$$= \frac{\sqrt{7}}{7\sqrt{7}^2}$$

$$= \frac{\sqrt{7}}{49}$$

$$(-2\sqrt{3})^2 = 2^2 \times \sqrt{3}^2$$

$$= 4 \times 3$$

$$= 12$$

$$-\sqrt{2}^6 = -(\sqrt{2^3})^2$$

$$= -2^3$$

$$= -8$$

$$B = \left(\frac{4}{3}\right)^2 - \left(-\frac{3}{2}\right)^{-2}$$

$$= \frac{16}{9} - \left(-\frac{2}{3}\right)^2$$

$$= \frac{16}{9} - \frac{4}{9}$$

$$= \frac{12}{9}$$

$$= \frac{4}{3}$$

;;

$$A = \left(\frac{\sqrt{7}}{3}\right)^4 \times \left(-\frac{\sqrt{7}}{3}\right)^{-4}$$

$$= \left(\frac{\sqrt{7}}{3}\right)^4 \times \left(-\frac{3}{\sqrt{7}}\right)^4$$

$$= \left(\frac{\sqrt{7}}{3} \times \frac{-3}{\sqrt{7}}\right)^4$$

$$= (-1)^4$$

$$= 1$$

;;

$$(3^{-1} + 2^{-1})^2 = \left(\frac{1}{3} + \frac{1}{2}\right)^2$$

$$= \left(\frac{2}{6} + \frac{3}{6}\right)^2$$

$$= \left(\frac{5}{6}\right)^2$$

$$= \frac{25}{36}$$

$$\begin{aligned}
 E &= \left[\left(\frac{5}{3} \right)^{-2} + \left(\frac{3}{5} \right)^2 \right]^{-1} \\
 &= \left[\left(\frac{3}{5} \right)^2 + \left(\frac{3}{5} \right)^2 \right]^{-1} \\
 &= \left[\frac{9}{25} + \frac{9}{25} \right]^{-1} \\
 &= \left[\frac{18}{25} \right]^{-1} \\
 &= \frac{25}{18}
 \end{aligned}$$

$$\begin{aligned}
 D &= \left[\left(\frac{25}{9} \right)^{-1} - \left(\frac{4}{5} \right)^2 \right]^{-1} \\
 &= \left[\frac{9}{25} - \frac{16}{25} \right]^{-1} \\
 &= \left[-\frac{7}{25} \right]^{-1} \\
 &= -\frac{25}{7}
 \end{aligned}$$

$$\begin{aligned}
 C &= \sqrt{10}^5 \times (-\sqrt{5})^3 \times \sqrt{10}^{-3} \times (-\sqrt{5})^{-5} \\
 &= \sqrt{10}^{5-3} \times (-\sqrt{5})^{3-5} \\
 &= \sqrt{10}^2 \times (-\sqrt{5})^{-2} \\
 &= \sqrt{10}^2 \times \left(-\frac{1}{\sqrt{5}} \right)^2 \\
 &= \left(\sqrt{10} \times \frac{1}{-\sqrt{5}} \right)^2 \\
 &= \left(-\frac{\sqrt{10}}{\sqrt{5}} \right)^2 \\
 &= \left(-\sqrt{\frac{10}{5}} \right)^2 \\
 &= \frac{10}{5} = 2
 \end{aligned}$$

$$\begin{aligned}
 e &= \left(-\frac{\sqrt{7}}{3} \right)^{120} \times \left(-\frac{3}{\sqrt{7}} \right)^{14} \\
 &= \left(-\frac{3}{\sqrt{7}} \right)^{-120} \times \left(-\frac{3}{\sqrt{7}} \right)^{14} \\
 &= \left(-\frac{3}{\sqrt{7}} \right)^{-120+14} \\
 &= \left(-\frac{3}{\sqrt{7}} \right)^{-106} \quad \text{و} \quad = \left(\frac{\sqrt{7}}{3} \right)^{106}
 \end{aligned}$$

$$\begin{aligned}
 f &= \sqrt{15}^{-15} \times \sqrt{3}^3 \times \sqrt{5}^7 \times \left(\frac{1}{\sqrt{5}} \right)^4 \\
 &= (\sqrt{5 \times 3})^{-15} \times \sqrt{3}^3 \times \sqrt{5}^7 \times \left(\frac{1}{\sqrt{5}} \right)^4 \\
 &= (\sqrt{5} \times \sqrt{3})^{-15} \times \sqrt{3}^3 \times \sqrt{5}^7 \times \sqrt{5}^{-4} \\
 &= \sqrt{5}^{-15} \times \sqrt{3}^{-15} \times \sqrt{3}^3 \times \sqrt{5}^7 \times \sqrt{5}^{-4} \\
 &= (\sqrt{5})^{-15+7-4} \times (\sqrt{3})^{-15+3} \\
 &= \sqrt{5}^{-12} \times \sqrt{3}^{-12} \\
 &= (\sqrt{5} \times \sqrt{3})^{-12} = \sqrt{15}^{-12}
 \end{aligned}$$

$$\begin{aligned}
 d &= \sqrt{6}^{-3} \times \left(\frac{1}{\sqrt{6}} \right)^{-8} \\
 &= \left(\frac{1}{\sqrt{6}} \right)^3 \times \left(\frac{1}{\sqrt{6}} \right)^{-8} \\
 &= \left(\frac{1}{\sqrt{6}} \right)^{3-8} \\
 &= \left(\frac{1}{\sqrt{6}} \right)^{-5} \quad \text{و} \quad = \sqrt{6}^5
 \end{aligned}$$

تمرين ② :

(1) - لكتب ما يلي على شكل قوة :

$$\begin{aligned}
 a &= -27 \\
 &= -3^3 \\
 &= (-3)^3
 \end{aligned}$$

$b = -25$ لا يمكن لأن : $-5^2 \neq (-5)^2$

$$\begin{aligned}
 c &= \left(\sqrt{2}^{-2} \times \frac{1}{\sqrt{2}^5} \right)^{-7} \\
 &= \left(\left(\frac{1}{\sqrt{2}} \right)^2 \times \left(\frac{1}{\sqrt{2}} \right)^5 \right)^{-7} \\
 &= \left(\left(\frac{1}{\sqrt{2}} \right)^{2+5} \right)^{-7} \\
 &= \left(\left(\frac{1}{\sqrt{2}} \right)^7 \right)^{-7} \\
 &= \left(\frac{1}{\sqrt{2}} \right)^{-49} \quad \text{و} \quad = \sqrt{2}^{49}
 \end{aligned}$$

(2) - لنكتب ما يلي على شكل قوة للعدد 10 :

$$\begin{aligned} A &= 10^{-15} \times 0,00001^2 \times \frac{1}{10000} \\ &= 10^{-15} \times (10^{-5})^2 \times 10^{-4} \\ &= 10^{-15} \times 10^{-10} \times 10^{-4} \\ &= 10^{-29} \end{aligned}$$

$$\begin{aligned} B &= \frac{(10^{-3})^{-2} \times 100^{-2} \times 0,002^2}{4 \times 10^{-4} \times 0,0001^{-3}} \\ &= \frac{10^6 \times (10^2)^{-2} \times (2 \times 10^{-3})^2}{4 \times 10^{-4} \times (10^{-4})^{-3}} \\ &= \frac{10^6 \times 10^{-4} \times 4 \times 10^{-6}}{4 \times 10^{-4} \times 10^{12}} \\ &= \frac{4}{4} \times \frac{10^{6-4-6}}{10^{-4+12}} \\ &= \frac{10^{-4}}{10^8} \\ &= 10^{-4-8} \\ &= 10^{-12} \end{aligned}$$

(3) - بسط ما يلي :

$$\begin{aligned} C &= \frac{4^3 \times 25^3 \times 10^{-4}}{5^6 \times 400^{-3} \times 10^2} \\ &= \frac{4^3 \times (5^2)^3 \times 10^{-4}}{5^6 \times (4 \times 10^2)^{-3} \times 10^2} \\ &= \frac{4^3 \times 5^6 \times 10^{-4}}{5^6 \times 4^{-3} \times 10^{-6} \times 10^2} = \frac{4^3 \times 5^6}{5^6 \times 4^{-3}} \times \frac{10^{-4}}{10^{-6} \times 10^2} \\ &= 4^{3+3} \times 5^{6-6} \times 10^{-4+6-2} \\ &= 4^6 \times 5^0 \times 10^0 \\ &= 4^6 \end{aligned}$$

$$\begin{aligned} D &= \frac{5 \times 10^{-7} + 1,5 \times 10^{-6} + 800 \times 10^{-8}}{0,15 \times 10^{12} + 85 \times 10^{10}} \\ &= \frac{5 \times 10^{-7} + 15 \times 10^{-7} + 80 \times 10^{-7}}{15 \times 10^{10} + 85 \times 10^{10}} \\ &= \frac{(5+15+80) \times 10^{-7}}{(15+85) \times 10^{10}} \\ &= \frac{100 \times 10^{-7}}{100 \times 10^{10}} \\ &= 10^{-7-10} \\ &= 10^{-17} \end{aligned}$$

تمرين ③ :

a و b عددان حقيقيان غير منعدمين بحيث : $a \neq 3$. لنسط ما يلي :

$$\begin{aligned} D &= \frac{(a^2)^{-2} \times (a^3)^{-3}}{(a^2)^{-3}} \\ &= \frac{a^{-4} \times a^{-9}}{a^{-6}} \\ &= a^{-4-9+6} \\ &= a^{-7} \end{aligned} \quad ; ; \quad \begin{aligned} C &= \frac{a^{-5} \times b^{-3} \times a^{-2}}{a^{-3} \times (b^{-2})^3} \\ &= \frac{a^{-5-2} \times b^{-3}}{a^{-3} \times b^{-6}} \\ &= \frac{a^{-7} \times b^{-3}}{a^{-3} \times b^{-6}} \\ &= a^{-7+3} \times b^{-3+6} \\ &= a^{-4} \times b^3 \end{aligned} \quad ; ; \quad \begin{aligned} B &= \frac{2a^5}{3a^4} \times \frac{a^{11}}{2a^2} \times \frac{a^3}{7a^{-3}} \\ &= \frac{2 \times a^{5+11+3}}{3 \times 2 \times 7 \times a^{4+2-3}} \\ &= \frac{a^{19}}{3 \times 7 \times a^3} \\ &= \frac{a^{19-3}}{21} \\ &= \frac{a^{16}}{21} \end{aligned} \quad ; ; \quad \begin{aligned} A &= \left[1 + \left(\frac{3-a}{1+a} \right)^{-1} \right]^{-1} \\ &= \left[1 + \frac{1+a}{3-a} \right]^{-1} \\ &= \left[\frac{3-a}{3-a} + \frac{1+a}{3-a} \right]^{-1} \\ &= \left[\frac{3-a+1+a}{3-a} \right]^{-1} \\ &= \left[\frac{4}{3-a} \right]^{-1} \\ &= \frac{3-a}{4} \end{aligned}$$

تمرين ④

(1) - لنحدد قيمة العدد الصحيح الطبيعي n بحيث :

$$\frac{9^{2n-1} \times 3^{n+1}}{27^{n+3}} = 81$$

لدينا :

$$\frac{9^{2n-1} \times 3^{n+1}}{27^{n+3}} = 3^4$$

$$\frac{(3^2)^{2n-1} \times 3^{n+1}}{(3^3)^{n+3}} = 3^4$$

$$\frac{3^{2(2n-1)+(n+1)}}{3^{3(n+3)}} = 3^4$$

$$3^{2(2n-1)+(n+1)-3(n+3)} = 3^4$$

و منه فإن :

$$2(2n-1)+(n+1)-3(n+3) = 4$$

$$4n-2+n+1-3n-9 = 4$$

$$4n+n-3n = 4+2-1+9$$

$$2n = 14$$

$$n = \frac{14}{2} = 7$$

(2) - أحسب ما يلي :

$$M = \frac{3^2 \times \left(\frac{1}{2}\right)^{-3} \times 5}{\left(\frac{120}{700}\right)^0 \times 3 \times \left(\frac{1}{5}\right)^{-1}}$$

$$= \frac{9 \times 2^3 \times 5}{1 \times 3 \times 5}$$

$$= \frac{9 \times 8 \times 5}{1 \times 3 \times 5}$$

$$= \frac{3 \times 8 \times 1}{1 \times 1 \times 1}$$

$$= 24$$

تمرين ⑤

(1) - لنحدد الكتابة العلمية للعدد a :

لدينا :

$$a = 2517,301 \times 10^{51}$$

$$= 2,517301 \times 10^3 \times 10^{51}$$

$$= 2,517301 \times 10^{54}$$

إذن الكتابة العلمية للعدد a هي : $2,517301 \times 10^{54}$.

(2) - لنحدد الكتابة العلمية للعدد b :

لدينا :

$$b = -0,000021 \times 10^{23}$$

$$= -2,1 \times 10^{-5} \times 10^{23}$$

$$= -2,1 \times 10^{-5+23}$$

$$= -2,1 \times 10^{18}$$

إذن الكتابة العلمية للعدد b هي : $-2,1 \times 10^{18}$.

(3) - لنحدد الكتابة العلمية للعدد c :

لدينا :

$$c = 113 \times 10^5 + 7,2 \times 10^7$$

$$= 113 \times 10^5 + 7,2 \times 10^2 \times 10^5$$

$$= (113 + 7,2 \times 10^2) \times 10^5$$

$$= (113 + 720) \times 10^5$$

$$= 833 \times 10^5$$

$$= 8,33 \times 10^2 \times 10^5$$

$$= 8,33 \times 10^7$$

إذن الكتابة العلمية للعدد c هي : $8,33 \times 10^7$.

(4) - لنحدد الكتابة العلمية للعدد d :

$$\begin{aligned}d &= 5 \times 10^{-3} + 3,2 \times 10^{-7} - 523 \times 10^{-5} \\&= 5 \times 10^{-3} + 3,2 \times 10^{-4} \times 10^{-3} - 523 \times 10^{-2} \times 10^{-3} \\&= (5 + 3,2 \times 10^{-4} - 523 \times 10^{-2}) \times 10^{-3} \\&= (5 + 0,00032 - 5,23) \times 10^{-3} \\&= 0,22968 \times 10^{-3} \\&= 2,2968 \times 10^{-1} \times 10^{-3} \\&= 2,2968 \times 10^{-4}\end{aligned}$$

إذن الكتابة العلمية للعدد d هي : $2,2968 \times 10^{-4}$.

(5) - لنحدد الكتابة العلمية للعدد e :
لدينا :

$$\begin{aligned}e &= \frac{3,2 \times 10^{-1} \times 5 \times (10^2)^3}{4 \times 10^{-2}} \\&= \frac{3,2 \times 5 \times 10^{-1} \times 10^6}{4 \times 10^{-2}} \\&= \frac{16 \times 10^{-1+6}}{4 \times 10^{-2}} \\&= \frac{16 \times 10^5}{4 \times 10^{-2}} \\&= \frac{16}{4} \times 10^{5+2} \\&= 4 \times 10^7\end{aligned}$$

إذن الكتابة العلمية للعدد e هي : 4×10^7 .

(5) - لنحدد الكتابة العلمية للعدد f :
لدينا :

$$\begin{aligned}f &= \frac{123 \times 10^{-3} + 7,2 \times 10^4}{2,5 \times 10^{-2}} \\&= \frac{0,123 + 72000}{2,5 \times 10^{-2}} \\&= \frac{72000,123}{2,5 \times 10^{-2}} \\&= \frac{7,2000123 \times 10^4}{2,5 \times 10^{-2}} \\&= \frac{7,2000123}{2,5} \times \frac{10^4}{10^{-2}} \\&= 2,88000492 \times 10^{4+2} \\&= 2,88000492 \times 10^6\end{aligned}$$

إذن الكتابة العلمية للعدد f هي : $2,88000492 \times 10^6$.

(2) - أثبت أن العدد : $K = 12^{100} \times \left(\frac{3}{2}\right)^{50} \times 6^{-149}$ صحيح طبيعي.

لدينا :

$$\begin{aligned}K &= 12^{100} \times \left(\frac{3}{2}\right)^{50} \times 6^{-149} \\&= (3 \times 2^2)^{100} \times \frac{3^{50}}{2^{50}} \times (3 \times 2)^{-149} \\&= 3^{100} \times 2^{200} \times 3^{50} \times 2^{-50} \times 3^{-149} \times 2^{-149} \\&= 3^{100+50-149} \times 2^{200-50-149} \\&= 3^1 \times 2^1 \\&= 6\end{aligned}$$

و بالتالي فإن K عدد صحيح طبيعي.