

حلول التمارين حول القوى

<p style="text-align: right;">(9)</p> $\begin{aligned} (2-3(2-3)^{-1})^{-1} &= [2-3(-1)^{-1}]^{-1} \\ &= \left[2-3\left(\frac{1}{-1}\right)\right]^{-1} \\ &= [2-3(-1)]^{-1} \\ &= [2+3]^{-1} \\ &= 5^{-1} \\ &= \frac{1}{5} \end{aligned}$ <p>إذن الإجابة الصحيحة هي $\frac{1}{5}$</p> <p>B = (3x-7)² A = (2x+3)² (10)</p> $\begin{aligned} (3x)^2 - 2 \times 3x \times 7 + 7^2 &= (2x)^2 + 2 \times 2x \times 3 + 3^2 \\ = 9x^2 - 42x + 49 &= 4x^2 + 12x + 9 \end{aligned}$ <p>D = (2x² + 5)(2x² - 5) C = \left(x - \frac{2}{3}\right)\left(x + \frac{2}{3}\right)</p> $\begin{aligned} = (2x^2)^2 - 5^2 &= x^2 - \left(\frac{2}{3}\right)^2 \end{aligned}$	<p>(1)</p> $\begin{aligned} (-5)^3 &= -125 \quad , \quad (-2)^5 = -32 \quad , \quad 2^3 = 8 \\ 0^{20} &= 0 \quad , \quad (-1)^{112} = 1 \quad , \quad 1^{75} = 1 \end{aligned}$ <p>(2)</p> $\begin{aligned} B &= [2 \times (-5)]^2 & A &= (-2)^3 \times (-3)^2 \\ &= (-10)^2 & &= -8 \times 9 \\ &= 100 & &= -72 \end{aligned}$ <p>C = [(-1)¹⁷ \times (-2)³]²</p> $\begin{aligned} &= [-1 \times (-8)]^2 \\ &= 8^2 \\ &= 64 \end{aligned}$ <p>D = [2 \times (-3)²] \times [4 \times (-5)²]²</p> $\begin{aligned} &= (2 \times (-3)^2) \times (4 \times (-5)^2)^2 \\ &= (2 \times 9) \times (4 \times 25)^2 \\ &= 18 \times 100^2 \\ &= 180000 \end{aligned}$
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$$= 4x^4 - 25 \quad = x^2 - \frac{4}{9}$$

$$x^2 + 8x + 16 = (x + 4)^2 \quad (11)$$

$$(3x - 2)^2 = 9x^2 - 12x + 4$$

$$4x^2 - \frac{25}{9} = \left(2x + \frac{5}{3}\right)\left(2x - \frac{5}{3}\right)$$

$$x^2 + x + \frac{1}{4} = \left(x + \frac{1}{2}\right)^2$$

$$B = x^2 - \frac{1}{2}x + \frac{1}{16} \quad A = 4a^2 - 49 \quad (12)$$

$$= x^2 - 2 \cdot \frac{1}{4} \cdot x + \left(\frac{1}{4}\right)^2 \quad = (2a)^2 - 7^2$$

$$= \left(x - \frac{1}{4}\right)^2 \quad = (2a + 7)(2a - 7)$$

$$C = (x^2 - 4) + (x - 2)(5x + 3)$$

$$= (x - 2)(x + 2) + (x - 2)(5x + 3)$$

$$= (x - 2)[(x + 2) + (5x + 3)]$$

$$= (x - 2)(6x + 5)$$

$$D = 3(x - 5)^2 - 2(x^2 - 25)$$

$$= 3(x - 5)(x - 5) - 2(x - 5)(x + 5)$$

$$= (x - 5)[3(x - 5) - 2(x + 5)]$$

$$= (x - 5)(3x - 15 - 2x - 10)$$

$$= (x - 5)(x - 25)$$

13 (أ) إذا كان n زوجيا فإن n+1 فردي،
ومنه :

$$A = (-1)^n + (-1)^{n+1}$$

$$= 1 + (-1)$$

$$= 0$$

$$B = 3^2(-1)^n - (-2)^2(-1)^{n+1}$$

$$= 3^2 \times 1 - (-2)^2(-1)$$

$$= 9 + 4$$

$$= 13$$

ب - إذا كان n فرديا فإن n+1 زوجي ومنه :

$$B = \left(\frac{-16 \times 3^2}{24 \times (-3)}\right)^{-1} \quad A = \left(\frac{2}{3}\right)^{-1} \times \left(\frac{-3}{4}\right)^{-1} \quad (3)$$

$$= \frac{24 \times (-3)}{-16 \times 3^2} \quad = \frac{3}{2} \times \frac{-4}{3}$$

$$= \frac{-8 \times 3 \times 3}{-8 \times 2 \times 3 \times 3} \quad = \frac{-4}{2}$$

$$= \frac{-1}{-2} = \frac{1}{2} \quad = -2$$

$$A = (-3)^5 \times (-3)^7 \times (-3)^{-11} \quad (4)$$

$$= (-3)^{5+7-11}$$

$$= (-3)^1$$

$$= -3$$

$$B = \frac{(-5)^2 \times (25)^{-3}}{5^3 \times (25)^{-2}}$$

$$\text{لاحظ أن } = \frac{(-5)^2 \times (5^2)^{-3}}{5^3 \times (5^2)^{-2}}$$

$$(-5)^2 = 5^2 \text{ و } 25 = 5^2$$

$$= \frac{5^2 \times 5^{-6}}{5^3 \times 5^{-4}}$$

$$= \frac{5^{2-6}}{5^{3-4}}$$

$$= \frac{5^{-4}}{5^{-1}}$$

$$= 5^{-4+1}$$

$$= 5^{-3}$$

$$= \frac{1}{5^3}$$

$$= \frac{1}{125}$$

$$A = \frac{a^2 b^3}{a^3 b^2} = \frac{a^2 b^2 b}{a^2 a b^2} = \frac{b}{a} \quad (5)$$

$$B = \frac{(2a^2 \times b^3)^3}{(3ab^4)^2} = \frac{2^3 a^6 b^9}{3^2 a^2 b^8} = \frac{8a^4 b}{9}$$

$$\begin{aligned} A &= (-1)^n + (-1)^{n+1} \\ &= -1 + 1 \\ &= 0 \end{aligned}$$

$$\begin{aligned} B &= 3^2(-1)^n - (-2)^2(-1)^{n+1} \\ &= 3^2 \times (-1) - (-2)^2(1) \\ &= -9 - 4 \\ &= -13 \end{aligned}$$

ملاحظة : في كلتا الحالتين $A = 0$ ونتيجتي B

متقابلتين.

(14) نبسط أولاً a و b باستعمال التعميل

$$\begin{aligned} a &= 2^{n-1} + 2^n + 2^{n+1} \\ &= 2^{n-1}(1 + 2 + 2^2) \\ &= 2^{n-1}(7) \\ &= 7 \times 2^{n-1} \end{aligned}$$

$$\begin{aligned} b &= 2^{n-2} + 2^{n-1} + 2^n \\ &= 2^{n-2}(1 + 2 + 2^2) \\ &= 2^{n-2}(7) \\ &= 7 \times 2^{n-2} \end{aligned}$$

$$\begin{aligned} a^2 &= (7 \times 2^{n-1})^2 \\ &= 7^2 \times (2^{n-1})^2 \\ &= 49 \times 2^{2n-2} \end{aligned}$$

$$\begin{aligned} b^2 &= (7 \times 2^{n-2})^2 \\ &= 7^2 \times (2^{n-2})^2 \\ &= 49 \times 2^{2n-4} \end{aligned}$$

$$\begin{aligned} c^2 &= (\sqrt{147} \times 2^{n-2})^2 \\ &= 147 \times 2^{2n-4} \end{aligned}$$

$$\begin{aligned} b^2 + c^2 &= 49 \times 2^{2n-4} + 147 \times 2^{2n-4} \quad \text{و منه} \\ &= (49 + 147) 2^{2n-4} \\ &= 196 \times 2^{2n-4} \\ &= 49 \times 4 \times 2^{2n-4} \\ &= 49 \times 2^2 \times 2^{2n-4} \\ &= 49 \times 2^{2n-2} \end{aligned}$$

لدينا $b^2 + c^2 = a^2$ إذن : المثلث ABC قائم الزاوية في A.

(15)

$$^{+1}.11^{3k+1}.5^{3k} + 539 = 7 \times 7^{3k}.11 \times 11^{3k}.5^{3k} + 7 \times 11 \times 7$$

$$\begin{aligned} C &= \frac{a^2 b^3}{(ab)^3} \div \left(\frac{a^2 b^4}{a^3 b^5} \right)^{-1} = \frac{a^2 b^3}{(ab)^3} \div \frac{a^3 b^5}{a^2 b^4} \\ &= \frac{a^2 b^3}{a^3 b^3} \times \frac{a^2 b^4}{a^3 b^5} = \frac{a^2 b^3 a^2 b^4}{a^3 b^3 a^3 b^5} = \frac{a^4 b^7}{a^6 b^8} = \frac{1}{a^2 b} \end{aligned}$$

$$A = (ab)^2 \left(\frac{a^2}{b^2} + \frac{b^2}{a^2} \right) \quad (6)$$

$$= a^2 b^2 \left(\frac{a^4 + b^4}{a^2 b^2} \right)$$

$$= a^4 + b^4$$

$$B = \frac{(ab^3)^2}{a^3 b^2} \left[\frac{a}{b^4} + \frac{a^2}{b^3} \right]$$

$$= \frac{a^2 b^6}{a^3 b^2} \left(\frac{a + a^2 b}{b^4} \right)$$

$$= \frac{a^2 b^6 (a + a^2 b)}{a^3 b^2 b^4}$$

$$= \frac{a^2 b^6 a(1 + ab)}{a^3 b^6}$$

$$= \frac{a^3 b^6 (1 + ab)}{a^3 b^6}$$

$$= 1 + ab$$

$$A = 2,3 \times 10^2 + 5,28 \times 10^{-1} \quad (7)$$

$$= 230 + 0,528$$

$$= 230,528$$

$$= 2,30528 \times 10^2$$

$$B = (53,27 \times 10^{-2}) \div 20$$

$$= 0,5327 \div 20$$

$$= 0,026635$$

$$= 2,6635 \times 10^{-2}$$

$$C = \frac{45 \times 10^{-2}}{12 \times 10^{-3}}$$

$$= \frac{45}{12} \times 10^{-2+3}$$

$$= \frac{15}{4} \times 10^1$$

$$= 3,75 \times 10$$

$$\begin{aligned}
 &= 7 \times 11 \times (7^{3k} \cdot 11^{3k} \cdot 5^{3k} + 7) \\
 &= 77(7 \times 7^{3k-1} \cdot 11^{3k} \cdot 5^{3k} + 7) \\
 &= 77 \times 7(7^{3k-1} \cdot 11^{3k} \cdot 5^{3k} + 1) \\
 &= 539(7^{3k-1} \cdot 11^{3k} \cdot 5^{3k} + 1)
 \end{aligned}$$

وبملاحظة أن $7^{3k-1} \cdot 11^{3k} \cdot 5^{3k}$ عدد فردي (جداء أعداد فردية) فإن $7^{3k-1} \cdot 11^{3k} \cdot 5^{3k} + 1$ عدد زوجي أي $Z) 7^{3k+1} \cdot 11^{3k+1} \cdot 5^{3k} + 539 = 539(2k')$
($k' \in \mathbb{Z}$)
 $= 1078k'$

إذن $7^{3k+1} \cdot 11^{3k+1} \cdot 5^{3k} + 539$ مضاعف للعدد 1078

$$(x^{-1} + y^{-1})^{-1} = \left(\frac{1}{x} + \frac{1}{y} \right)^{-1} \quad (8)$$

$$= \left(\frac{y+x}{xy} \right)^{-1}$$

$$= \left(\frac{x+y}{xy} \right)^{-1}$$

$$= \frac{xy}{x+y}$$

إذن الإجابة الصحيحة هي $\frac{xy}{x+y}$