

$$\lim_{x \rightarrow -\infty} \frac{-3x^6 + 2x^2 + 1}{x^3 + 3x - 1} \text{ و } \lim_{x \rightarrow +\infty} \frac{2x^5 + 3x^3 + x}{-5x^5 + 2x - 1} \text{ و } \lim_{x \rightarrow -\infty} -2x^4 + 3x^3 + x - 1 \text{ و } \lim_{x \rightarrow 2} 3 + x - 3x^2$$

$$\lim_{x \rightarrow 2^+} \frac{-2x + 1}{x^2 - x - 2} \text{ و } \lim_{x \rightarrow 1^+} \frac{-2x + 1}{x^2 - 3x + 2} \text{ و } \lim_{x \rightarrow 1} \frac{2x^2 - 5x + 3}{x^2 + 2x - 3} \text{ و } \lim_{x \rightarrow +\infty} \frac{7x^4 + x^2 + 1}{-x^8 - 2x - 1}$$

$$\text{ و } \lim_{x \rightarrow -\infty} \sqrt{x^2 + x} + x \text{ و } \lim_{x \rightarrow +\infty} \sqrt{x^2 + x} - x \text{ و } \lim_{x \rightarrow 1} \frac{\sqrt{2x-1} - 1}{x-1} \text{ و } \lim_{x \rightarrow 2^-} \frac{-2x + 1}{x^2 - x - 2}$$

$$\lim_{x \rightarrow 0} \frac{\sqrt{x+1} - 1}{\tan x} \text{ و } \lim_{x \rightarrow \frac{\pi}{2}} \frac{1}{\tan^2 x + 2} \text{ و } \lim_{x \rightarrow 0} \frac{\sin 5x}{\sin x} \text{ و } \lim_{x \rightarrow +\infty} \frac{\sqrt{x + \sqrt{x}} + \sqrt{x}}{\sqrt{x+1}} \text{ و } \lim_{x \rightarrow +\infty} \frac{x}{x - 2\sqrt{x+1}}$$

$$\lim_{x \rightarrow \frac{\pi}{2}^+} \frac{\cos x}{1 - \sin x} \text{ و } \lim_{x \rightarrow 0} \frac{\cos 3x - \cos x}{\sin 3x + \sin x}$$

الجواب

نحدد النهايات

$$\lim_{x \rightarrow -\infty} -2x^4 + 3x^3 + x - 1 = \lim_{x \rightarrow -\infty} -2x^4 = -\infty \quad \lim_{x \rightarrow 2} 3 + x - 3x^2 = -7 \quad *$$

$$\lim_{x \rightarrow +\infty} \frac{2x^5 + 3x^3 + x}{-5x^5 + 2x - 1} = \lim_{x \rightarrow +\infty} \frac{2x^5}{-5x^5} = -\frac{2}{5} \quad *$$

$$\lim_{x \rightarrow -\infty} \frac{-3x^6 + 2x^2 + 1}{x^3 + 3x - 1} = \lim_{x \rightarrow -\infty} \frac{-3x^6}{x^3} = \lim_{x \rightarrow -\infty} -3x^3 = +\infty \quad *$$

$$\lim_{x \rightarrow +\infty} \frac{7x^4 + x^2 + 1}{-x^8 - 2x - 1} = \lim_{x \rightarrow +\infty} \frac{7x^4}{-x^8} = \lim_{x \rightarrow +\infty} \frac{-7}{x^4} = 0 \quad *$$

$$\lim_{x \rightarrow 1} \frac{2x^2 - 5x + 3}{x^2 + 2x - 3} = \lim_{x \rightarrow 1} \frac{(x-1)(2x-3)}{(x-1)(x+3)} = \lim_{x \rightarrow 1} \frac{2x-3}{x+3} = -\frac{1}{4} \quad *$$

$$\lim_{x \rightarrow 1^+} \frac{-2x+1}{x^2-3x+2} \quad \text{نحدد} *$$

x	$-\infty$	1	2	$+\infty$
$x^2 - 3x + 2$		$+$	0	$-$
		0	$+$	

$$\lim_{x \rightarrow 1^+} \frac{-2x+1}{x^2-3x+2} = +\infty \text{ ومنه } \lim_{x \rightarrow 1^+} -2x+1 = -1 \quad \lim_{x \rightarrow 1^+} x^2 - 3x + 2 = 0^-$$

* لدينا $\lim_{x \rightarrow 2} -2x + 1 = -3$ و $\lim_{x \rightarrow 2} x^2 - x - 2 = 0$

x	$-\infty$	-1	2	$+\infty$
$x^2 - x - 2$		$+$	0	$-$

ومنه $\lim_{x \rightarrow 2^-} x^2 - x - 2 = 0^-$ و $\lim_{x \rightarrow 2^+} x^2 - x - 2 = 0^+$

إذن $\lim_{x \rightarrow 2^+} \frac{-2x + 1}{x^2 - x - 2} = -\infty$; $\lim_{x \rightarrow 2^-} \frac{-2x + 1}{x^2 - x - 2} = +\infty$

$$\lim_{x \rightarrow 1} \frac{\sqrt{2x-1}-1}{x-1} = \lim_{x \rightarrow 1} \frac{2x-2}{(x-1)(\sqrt{2x-1}+1)} = \lim_{x \rightarrow 1} \frac{2}{\sqrt{2x-1}+1} = 1 \quad *$$

$$\lim_{x \rightarrow +\infty} \sqrt{x^2+x} - x = \lim_{x \rightarrow +\infty} \frac{x}{\sqrt{x^2+x}+x} = \lim_{x \rightarrow +\infty} \frac{x}{\sqrt{x^2\left(1+\frac{1}{x}\right)}+x} = \lim_{x \rightarrow +\infty} \frac{x}{|x|\sqrt{1+\frac{1}{x}}+x} \quad \text{لدينا} \quad *$$

و حيث x تؤول إلى $+\infty$ فان x موجبة ومنه $|x| = x$

$$\lim_{x \rightarrow +\infty} \sqrt{x^2+x} - x = \lim_{x \rightarrow +\infty} \frac{x}{x\left(\sqrt{1+\frac{1}{x}}+1\right)} = \lim_{x \rightarrow +\infty} \frac{1}{\sqrt{1+\frac{1}{x}}+1} = \frac{1}{2} \quad \text{ومنه}$$

$$\lim_{x \rightarrow -\infty} \sqrt{x^2+x} + x = \lim_{x \rightarrow -\infty} \frac{x}{\sqrt{x^2+x}-x} = \lim_{x \rightarrow -\infty} \frac{x}{\sqrt{x^2\left(1+\frac{1}{x}\right)}-x} = \lim_{x \rightarrow -\infty} \frac{x}{|x|\sqrt{1+\frac{1}{x}}-x} \quad *$$

و حيث x تؤول إلى $-\infty$ فان x سالبة ومنه $|x| = -x$

$$\lim_{x \rightarrow -\infty} \sqrt{x^2+x} + x = \lim_{x \rightarrow -\infty} \frac{x}{-x\sqrt{1+\frac{1}{x}}-x} = \lim_{x \rightarrow -\infty} \frac{1}{-\sqrt{1+\frac{1}{x}}-1} = -\frac{1}{2} \quad \text{إذن}$$

$$\lim_{x \rightarrow +\infty} \frac{x}{x-2\sqrt{x+1}} = \lim_{x \rightarrow +\infty} \frac{x}{x-2\sqrt{x^2\left(\frac{1}{x}+\frac{1}{x^2}\right)}} = \lim_{x \rightarrow +\infty} \frac{x}{x-2|x|\sqrt{\frac{1}{x}+\frac{1}{x^2}}} \quad *$$

و حيث x تؤول إلى $+\infty$ فان x موجبة ومنه $|x| = x$

$$\lim_{x \rightarrow +\infty} \frac{x}{x-2\sqrt{x+1}} = \lim_{x \rightarrow +\infty} \frac{x}{x\left(1-2\sqrt{\frac{1}{x}+\frac{1}{x^2}}\right)} = \lim_{x \rightarrow +\infty} \frac{1}{1-2\sqrt{\frac{1}{x}+\frac{1}{x^2}}} = 1$$

$$\lim_{x \rightarrow +\infty} \frac{\sqrt{x}+\sqrt{x}+\sqrt{x}}{\sqrt{x+1}} = \lim_{x \rightarrow +\infty} \frac{\sqrt{x\left(1+\frac{1}{\sqrt{x}}\right)}+\sqrt{x}}{\sqrt{x+1}} = \lim_{x \rightarrow +\infty} \frac{\sqrt{x}\left(\sqrt{1+\frac{1}{\sqrt{x}}}+1\right)}{\sqrt{x}\sqrt{1+\frac{1}{x}}} = \lim_{x \rightarrow +\infty} \frac{\sqrt{1+\frac{1}{\sqrt{x}}}+1}{\sqrt{1+\frac{1}{x}}} = 2 \quad *$$

$$\lim_{x \rightarrow 0} \frac{\sin 5x}{\sin x} = \lim_{x \rightarrow 0} \frac{x}{\sin x} \times \frac{\sin 5x}{5x} \times 5 \quad \text{لدينا} \quad *$$

$$\lim_{x \rightarrow 0} \frac{x}{\sin x} = 1 \quad \text{ومنه} \quad \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \quad \text{نعلم أن}$$

$$\lim_{x \rightarrow 0} \frac{\sin 5x}{\sin x} = 1 \times 1 \times 5 = 5 \quad \text{فان} \quad \lim_{x \rightarrow 0} \frac{\sin 5x}{5x} = 1 \quad \text{وحيث}$$

$$\lim_{x \rightarrow \frac{\pi}{2}} \frac{1}{\tan^2 x + 2} = 0 \quad \text{ومنه} \quad \lim_{x \rightarrow \frac{\pi}{2}} \tan^2 x = +\infty \quad \text{لدينا} \quad *$$

$$\lim_{x \rightarrow 0} \frac{\sqrt{x+1}-1}{\tan x} = \lim_{x \rightarrow 0} \frac{x}{\tan x} \times \frac{1}{\sqrt{x+1}+1} \quad *$$

$$\lim_{x \rightarrow 0} \frac{x}{\tan x} = 1 \quad \text{ومنه} \quad \lim_{x \rightarrow 0} \frac{\tan x}{x} = 1 \quad \text{نعلم أن}$$

$$\lim_{x \rightarrow 0} \frac{\sqrt{x+1}-1}{\tan x} = 1 \times \frac{1}{2} = \frac{1}{2} \quad \text{إذن} \quad \lim_{x \rightarrow 0} \sqrt{x+1}+1 = 2 \quad \text{لدينا}$$

$$\lim_{x \rightarrow 0} \frac{\cos 3x - \cos x}{\sin 3x + \sin x} = \lim_{x \rightarrow 0} \frac{-2 \sin 2x \cdot \sin x}{2 \sin 2x \cdot \cos x} = \lim_{x \rightarrow 0} -\tan x = 0 \quad *$$

$$\lim_{x \rightarrow \frac{\pi}{2}^+} \frac{\cos x}{1 - \sin x} = \lim_{x \rightarrow \frac{\pi}{2}^+} \frac{(1 + \sin x) \cos x}{1 - \sin^2 x} = \lim_{x \rightarrow \frac{\pi}{2}^+} \frac{1 + \sin x}{\cos x} = \text{لدينا} \quad *$$

$$\forall x \in \left] -\pi; \frac{\pi}{2} \right[\quad \cos x < 0 \quad \text{لأن} \quad \lim_{x \rightarrow \frac{\pi}{2}^+} \cos x = 0^- \quad \text{و} \quad \lim_{x \rightarrow \frac{\pi}{2}^+} 1 + \sin x = 2 \quad \text{لدينا}$$

$$\lim_{x \rightarrow \frac{\pi}{2}^+} \frac{\cos x}{1 - \sin x} = \lim_{x \rightarrow \frac{\pi}{2}^+} \frac{1 + \sin x}{\cos x} = -\infty \quad \text{ومنه}$$

تمرين 2

$$\text{بين أن} \quad \lim_{x \rightarrow 0} x \sin \frac{1}{x} = 0$$

الجواب

$$\forall x \in \mathbb{R}^* \quad \left| x \sin \frac{1}{x} \right| \leq |x| \quad \text{ومنه} \quad \forall x \in \mathbb{R}^* \quad -1 \leq \sin \frac{1}{x} \leq 1 \quad \text{لدينا}$$

$$\lim_{x \rightarrow 0} x \sin \frac{1}{x} = 0 \quad \text{فان} \quad \lim_{x \rightarrow 0} |x| = 0$$

تمرين 3

$$g(x) = \frac{\sqrt{2 - \cos x} - 1}{x^2} \quad \text{نعتبر } g \text{ دالتين عددية للمتغير حقيقي } x \text{ حيث}$$

$$\lim_{x \rightarrow 0} g(x) \quad \text{حدد} \quad -1$$

$$\lim_{x \rightarrow +\infty} g(x) \quad \text{واستنتج} \quad \forall x \in \mathbb{R}^* \quad \left| g(x) \right| \leq \frac{1}{x^2} \quad -3 \text{ بين أن}$$

الجواب

1- نحدد $\lim_{x \rightarrow 0} g(x)$

$$\lim_{x \rightarrow 0} g(x) = \lim_{x \rightarrow 0} \frac{\sqrt{2 - \cos x} - 1}{x^2} = \lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} \times \frac{1}{\sqrt{2 - \cos x} + 1}$$

$$\lim_{x \rightarrow 0} \frac{1}{\sqrt{2 - \cos x} + 1} = \frac{1}{2} \quad \text{ومنه} \quad \lim_{x \rightarrow 0} 2 - \cos x = 1 \quad \text{لدينا}$$

$$\lim_{x \rightarrow 0} g(x) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} \quad \text{فان} \quad \lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} = \frac{1}{2} \quad \text{وحيث}$$

$$2- \text{ نبين أن } \forall x \in \mathbb{R}^* \quad |g(x)| \leq \frac{1}{x^2}$$

$$\forall x \in \mathbb{R}^* \quad -1 \leq \cos x \leq 1 \Leftrightarrow 1 \leq 2 - \cos x \leq 3 \Leftrightarrow 0 \leq \sqrt{2 - \cos x} - 1 \leq \sqrt{3} - 1 < 1$$

$$\forall x \in \mathbb{R}^* \quad |g(x)| = \left| \frac{\sqrt{2 - \cos x} - 1}{x^2} \right| = \frac{\sqrt{2 - \cos x} - 1}{x^2}$$

$$\forall x \in \mathbb{R}^* \quad 0 \leq \frac{\sqrt{2 - \cos x} - 1}{x^2} < \frac{1}{x^2} \quad \text{ومنه} \quad \forall x \in \mathbb{R}^* \quad 0 \leq \sqrt{2 - \cos x} - 1 < 1 \quad \text{لدينا}$$

$$\forall x \in \mathbb{R}^* \quad |g(x)| \leq \frac{1}{x^2} \quad \text{إذن}$$

نستنتج $\lim_{x \rightarrow +\infty} g(x)$

$$\lim_{x \rightarrow +\infty} g(x) = 0 \quad \text{ومنه} \quad \lim_{x \rightarrow +\infty} \frac{1}{x^2} = 0 \quad \text{و} \quad \forall x \in \mathbb{R}^* \quad |g(x)| \leq \frac{1}{x^2} \quad \text{لدينا}$$

تمارين غير محلولة

تمرين 1

$$\lim_{x \rightarrow -2} \frac{3x^2 - 12}{x + 2} ; \quad \lim_{x \rightarrow -3} \sqrt{x^2 - 3x} \quad \text{حدد} \quad \lim_{x \rightarrow -1} \frac{3x^2 - 2x}{x + 2} ; \quad \lim_{x \rightarrow -2} 2x^2 - 6x - 1$$

$$\lim_{x \rightarrow 0} \frac{x}{\sqrt{x+1} - 1} ; \quad \lim_{x \rightarrow 1} \frac{x^3 + x - 2}{2x^2 - x - 1} \quad \lim_{x \rightarrow 1} \frac{x^2 + x - 2}{2x^2 + x - 3} ; \quad \lim_{x \rightarrow 3} \frac{x - 3}{x^2 - 2x - 3}$$

$$\lim_{x \rightarrow 3} \frac{\sqrt{x+1} - 2}{\sqrt{x} - 2 - 1} ; \quad \lim_{x \rightarrow 0^+} \frac{\sqrt{x+4} - 2}{\sqrt{x}} \quad \lim_{x \rightarrow 1} \frac{\sqrt{2x^2 - 1} - x}{x - 1} ; \quad \lim_{x \rightarrow 2} \frac{1 - \sqrt{x-1}}{x - 2}$$

تمرين 2

نعتبر f دالة عددية

أدرس نهاية f على يمين ويسار x_0 واستنتج هل f تقبل نهاية في x_0

في الحالتين التاليتين

$$x_0 = 1 \quad \begin{cases} f(x) = \frac{(x-1)^2}{|x-1|} & x \neq 1 \\ f(1) = 2 \end{cases} \quad \text{ب-} \quad x_0 = 2 \quad \begin{cases} f(x) = \frac{-4+x^2}{x-2} & x > 2 \\ f(x) = \sqrt{x^2+12} & x \leq 2 \end{cases} \quad \text{أ-}$$

تمرين 3

$$\lim_{x \rightarrow 0} \frac{\cos x - 1}{x} ; \lim_{x \rightarrow 0} \frac{\tan 5x}{\sin 2x} ; \lim_{x \rightarrow 0} \frac{\sin 2x}{\sin 3x} \quad \lim_{x \rightarrow 0} \frac{\sin^2 x}{3x^2} , \lim_{x \rightarrow 0} \frac{\sin 2x}{5x} ; \lim_{x \rightarrow \frac{\pi}{2}} \frac{\sin 2x}{1 + \sin x} \quad \text{حدد}$$

$$\lim_{x \rightarrow 0} \frac{\sin 3x + \sin x}{\sin 5x - \sin x} \quad \lim_{x \rightarrow \frac{\pi}{4}} \frac{\cos x + \sin x}{x + \frac{\pi}{4}} \quad \lim_{x \rightarrow \frac{\pi}{3}} \frac{\sqrt{3} \cos x - \sin x}{x - \frac{\pi}{3}} ; \lim_{x \rightarrow 0} \frac{\sqrt{x+1} - 1}{\tan x}$$

$$\lim_{x \rightarrow 1} \frac{\sin \pi x}{x-1} \quad \lim_{x \rightarrow \frac{\pi}{4}} \frac{1 - \sqrt{2} \cos x}{1 - \sqrt{2} \sin x} \quad \lim_{x \rightarrow 0} \frac{\sin 3x}{\sin 5x + \sin 3x} \quad \lim_{x \rightarrow 0} \frac{\cos x - \cos 2x}{\sin x + \sin 2x}$$

$$\lim_{x \rightarrow \frac{\pi}{2}} (1 - \sin x) \tan x \quad \lim_{x \rightarrow 0} \frac{1 - \cos \sqrt{x}}{\sin x} \quad \lim_{x \rightarrow 2} \frac{\sin(x^2 - 2x)}{x - 2}$$

تمرين 4

$$\lim_{x \rightarrow 3^+} \frac{x^2 - 2x}{x - 3} \quad \lim_{x \rightarrow 2^-} \frac{x^2 - 2x}{|x - 2|} ; \lim_{x \rightarrow 1^+} \frac{2x^2 - 2}{\sqrt{x} - 1} \quad \text{حدد}$$

$$\lim_{x \rightarrow 1} \frac{x\sqrt{x} - 1}{x^2 - 1} \quad \lim_{x \rightarrow 1^-} \frac{\sqrt{1 - x^2}}{x - 1} \quad \lim_{x \rightarrow -2} \frac{x^2 + 3x}{x^2 - x - 6} \quad \lim_{x \rightarrow -2^+} \frac{x^2 + 3x}{x^2 - x - 6} \quad \lim_{x \rightarrow 3^-} \frac{x^2 - 2x}{x - 3}$$

تمرين 5

$$\lim_{x \rightarrow \pm\infty} \frac{-3x^6 - 2x}{x + 2} ; \lim_{x \rightarrow \pm\infty} \frac{3x^2 - 5x}{-6x^4 - 2x} \quad \lim_{|x| \rightarrow +\infty} 5x^3 - 3x ; \lim_{|x| \rightarrow +\infty} -4x^2 - 6x - 1 \quad \text{حدد}$$

$$\lim_{x \rightarrow +\infty} \frac{\sqrt{x+1} - 2}{x} ; \lim_{x \rightarrow +\infty} \frac{-2x}{\sqrt{x+1}} \quad \lim_{x \rightarrow +\infty} \sqrt{x+1} - x ; \lim_{x \rightarrow +\infty} \sqrt{x+1} + x$$

$$(x = \frac{1}{t} \text{ نضع}) \lim_{x \rightarrow +\infty} x^2 \left(1 - \cos \frac{1}{x}\right) \quad \lim_{x \rightarrow +\infty} \sqrt{x + \sqrt{x}} - \sqrt{x} \quad \lim_{x \rightarrow +\infty} \frac{\sqrt{x^2 + 1} - x}{\sqrt{x^2 - 1} - \sqrt{x^2 + 2}}$$

$$\lim_{x \rightarrow +\infty} x \sin x + 2x$$

تمرين 6

حدد مجموعة تعريف الدالة f و أحسب النهايات عند محددات D_f

$$f(x) = \frac{x^2 + 3x + 2}{x^2 - 3x + 4} \quad \text{ب-} \quad f(x) = 2x^2 - 3x - 1$$

$$f(x) = 2x - \sqrt{x^2 + 5x + 6} \quad \text{د-} \quad f(x) = \frac{x+1}{\sqrt{x-2}} \quad \text{ج-}$$

تمرين 7

$$\lim_{x \rightarrow -2^-} \frac{5x^2 - 3x}{x^2 - 3x - 10} \quad \lim_{x \rightarrow 2} \frac{\sqrt{x^2 - x} - \sqrt{2x - 2}}{x^2 - x - 2} \quad \lim_{x \rightarrow -\infty} \sqrt{x^2 - 3x} + x - 1 \quad \text{أحسب النهايات}$$

$$\lim_{x \rightarrow \frac{\pi}{2}^+} \frac{\cos 2x}{1 - \sin x} \quad \lim_{x \rightarrow 0} \frac{\cos x - \sqrt{1 + \sin^2 x}}{x^2} \quad \lim_{x \rightarrow +\infty} \frac{\sqrt{x-2} - \sqrt{x}}{\sqrt{x+2} + \sqrt{x}}$$

$$\lim_{x \rightarrow 0} \frac{\sin 2x - 2 \sin x}{x^3} \quad \lim_{x \rightarrow \frac{\pi}{4}} \frac{\cos x - \sin x}{1 - \sqrt{2} \cos x} \quad \lim_{x \rightarrow 0} \frac{1 - \cos^2(x^2 - x)}{x^2}$$

